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Entrepreneurial finance and risk in portfolio management: Drawdown-based systematic exposure in European equities

***Ewa Feder-Sempach*[1], *Piotr Szczepocki*[2], *Stan Uryasev*[3]**

Abstract

PURPOSE: *The objective of the article is to calculate new systematic risk measures – Expected Regret of Drawdown (ERoD) Beta and Conditional Drawdown-at-Risk (CDaR) Beta – that account for the drawdowns (a decline in the value from a peak to a subsequent low) of the STOXX Europe 600 index in the period 2004–2024 across European countries and industries. These drawdown betas show how the securities behaved during market drawdowns, especially during the Great Financial Crisis and the COVID-19 crisis.* ***METHODOLOGY:*** *ERoD and CDaR Beta are new dynamic portfolio performance risk characteristics measuring portfolio drawdowns. Similar to the standard beta, the drawdown betas relate the expected return of an asset to the expected return of the market but are based on the concept of drawdowns.* ***FINDINGS:*** *The numerical results show that drawdown beta directly measures the performance of the securities during drawdowns. The drawdown betas have negative values, suggesting a safe-haven property for certain companies during financial crises, whereas the standard beta is positive. We demonstrate that the drawdown beta is a useful tool for portfolio construction and can indicate the different performance of Healthcare, Technology, and small-value companies during the COVID-19 crisis. It can imply that those securities can provide a hedge during a similar crisis.* ***IMPLICATIONS:*** *If the negative drawdown beta is identified, it could be used as insurance to lessen risk or to perform well in the event of a crisis in active management strategies.* ***ORIGINALITY & VALUE:*** *The drawdown beta is an innovative dynamic portfolio performance risk measure used on the European stock market data across an extended period (2004–2024). We investigated the impact of different factors on the resilience of the company during drawdowns, including firm-size deciles, countries, and industries, providing valuable insights into the relationship between company size and resilience during downturns. The drawdown beta framework delivers a powerful tool for both academics and practitioners seeking to assess asset resilience during market drawdowns.*

Keywords: *drawdown beta, Conditional Drawdown-at-Risk (CDaR) beta, Expected Regret of Drawdown (ERoD) beta, downside risk, systematic risk, Capital Asset Pricing Model (CAPM), portfolio risk management, safe-haven assets, financial crises, European equity markets*

INTRODUCTION

The relationship between risk and expected return is a significant challenge in financial theory. The traditional Capital Asset Pricing Model (CAPM) explains how to measure risk and the relationship between expected return and risk (Sharpe, 1964; Lintner, 1965; Mossin, 1966). CAPM shows that investors are rewarded for taking on non-diversifiable

1 Ewa Feder-Sempach, Assistant Professor, University of Lodz, Faculty of Economics and Sociology, Department of International Economics, 90-255 Lodz, 3/5 POW Street, Poland, e-mail: ewa.feder@uni.lodz.pl (https://orcid.org/0000-0001-9586-1417).
2 Piotr Szczepocki, Assistant Professor, University of Lodz, Faculty of Economics and Sociology, Department of Statistical Methods, 90-255 Lodz, 3/5 POW Street, Poland, e-mail: piotr.szczepocki@uni.lodz.pl (https://orcid.org/0000-0001-8377-3831).
3 Stan Uryasev, Full Professor, Stony Brook University, Department of Applied Mathematics and Statistics, Stony Brook, NY 11794, USA, e-mail: stanislav.uryasev@stonybrook.edu (https://orcid.org/0000-0001-6950-3966).

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risk (systematic risk), which is measured by beta. Beta is the tendency of the stock's return to respond to changes in the market, represented by the stock market index. Beta coefficients are a fundamental concept in finance, particularly within the realm of portfolio management, as they serve as a key measure of an asset's riskiness.

While the single-period CAPM uses beta to explain returns on risky assets, beta alone cannot capture all variations in expected returns. The standard approach to estimate beta is to estimate covariances and variances from a time series of historical stock returns. However, this approach faces the problem of time-varying beta coefficients (e.g., Blume, 1975; Ferson and Harvey, 1991; and Groenewold and Fraser, 1999). Empirical studies across international markets, including the US (Fabozzi and Francis, 1978; Feder-Sempach et al., 2023) and Europe (Wells, 1994; Chauveau and Maillet, 1998), have demonstrated the instability of the beta parameter. Beta is mainly evaluated over relatively short periods (typically five years) because the CAPM's performance tends to be better over shorter periods, likely due to beta's instability over time. This phenomenon may stem from a conceptual limitation of the standard CAPM risk measure, which fails to distinguish between losses and gains within a portfolio. A new approach, represented by the drawdown beta, could better assess portfolio performance during market drawdowns.

Various risk measures have been proposed as alternatives to variance, including the latest drawdown-based risk measures, the Conditional Drawdown-at-Risk beta (CDaR beta), introduced by Zabarankin et al. (2014), and the Expected Regret of Drawdown Beta (ERoD beta), proposed by Ding and Uryasev (2022). Both metrics, like the standard or traditional beta, relate the returns of an asset to the returns of the market but are based on the concept of drawdowns: the decline in the value of an asset from a peak to a subsequent low. The CDaR and ERoD Betas differ in their approach to determining which drawdowns to include. The former is based on a percentage of worst-case market drawdowns, while the latter is based on drawdowns with a known threshold.

The main aim of the paper is to examine standard beta and the new drawdown betas during stock market drawdowns in Europe, applying the methodologies proposed by Zabarankin et al. (2014) and Ding and Uryasev (2022). To show the impact of unprecedented events – the Great Financial Crisis (GFC), the COVID-19 crisis, and Russia's invasion of Ukraine – the sample covers two 10-year subperiods: 2004–2014 and 2014-2024, with March 1, 2014, serving as the cutoff date. The drawdown beta evaluates portfolio performance, accounting for the drawdowns of the STOXX Europe 600 index across countries, industries, and company sizes, to differentiate between the causes and the nature of the crisis.

The research sample comprises companies in the STOXX Europe 600 index, listed in both euros and in their respective domestic currencies. This index comprises 600 components from European countries, covering large-, mid-, and small-capitalisation companies. The index provides coverage across country and industry allocations, offering a portfolio of developed European economies and replicating almost 90% of the underlying investable market.

This paper contributes to the literature in three significant ways. First, it assesses the two risk measures (CDaR and ERoD Betas) over a 20-year horizon, including two ten-year subperiods. The drawdown periods are defined differently based on their respective formulas. Second, it analyses these risk measures in different countries, industries and market capitalization deciles to illustrate the impact of the GFC and COVID-19 market turmoil. Third, by comparing the standard beta and drawdown betas, the aim is to supplement the existing portfolio management literature with a diversification perspective. The main research hypothesis is that drawdown beta is much more sensitive to market distress and can exhibit negative values, potentially working as a downside protection measure indicator, unlike the standard beta, which is positive. A downside protection measure indicator not based on correlation is similar to the notion of safe-haven assets, which maintain or increase their value during periods of market turmoil, protecting investors when most other assets decline. The above-mentioned sensitivity to crises could be effectively leveraged in active portfolio management strategies in European stock markets amid market volatility.

The remainder of the article is structured as follows: Section two provides a literature review, and Section three presents the research sample and methodology. Section four discusses the empirical results, and the last section concludes the paper.

LITERATURE REVIEW

The standard beta is a key indicator of stock performance in portfolio management, measuring the relationship between the expected return of a security and the expected excess return of a market index. According to the classical CAPM formula, this relationship is based on three implications: (1) expected returns on all assets are linearly related to their

betas, (2) the beta premium is positive, and (3) the beta premium equals the expected market return minus the risk-free rate (Fama and French, 2004).

Early empirical tests of the CAPM revealed that beta estimates for individual assets are imprecise and that regression residuals exhibit common sources of variation, such as industry effects in average returns. To improve beta estimation, researchers shifted their focus from individual securities to portfolios. For example, Blume (1970, 1975), Friend and Blume (1973), and Black, Jensen, and Scholes (1972) demonstrated that beta estimates for diversified portfolios are more precise than those for individual securities. Vasicek (1973) proposed a method of adjusting historical betas towards the average beta, modifying each estimate based on its sampling error. In other words, an estimate is considered more reliable if it predicts stock returns more accurately than simply assuming all stocks have a beta of one. Fama and MacBeth (1973) addressed inference problems caused by correlated residuals in cross-sectional regressions. Instead of using a single cross-sectional regression of average monthly returns on beta estimates, they proposed conducting month-by-month cross-sectional regressions of monthly returns on estimated betas.

The central question is whether market betas can explain expected returns. The Sharpe-Lintner and Black (1972) version of the CAPM implies that differences in expected return across securities and portfolios are entirely explained by differences in market beta. No other variables contribute to explaining expected returns – highlighting the prominent role of beta in modern financial theory. The risk premium for beta is assumed to be positive, and under these conditions, the model provides a good description of expected stock returns.

Furthermore, it is vital to know if the relationship between beta and return holds under different stock market conditions, such as bull and bear markets. Several studies have examined this using individual securities, including Fabozzi and Francis (1977, 1979), Clinball et al. (1993), Kim and Zumwalt (1979), and Dębski et al. (2016). Most of these studies find that the beta parameter varies with different market conditions. For example, Spiceland et al. (1983) observed that the parameters exhibit nonstationarity during market advances and market declines for certain stock groups. For instance, parameters of stocks in high-risk and low-risk schemes behave differently, suggesting they are affected by the alternating forces of bull and bear market conditions. Wiggins (1992) demonstrated that conditioning beta on the sign of the market risk premium provides a better explanation of monthly cross-sectional returns, on average. These studies commonly used the Dual Beta Market (DBM) model and t-tests and F-tests, along with up- and down-market definitions of bull and bear markets, to investigate this relationship. Notably, there is no single definition of a bull or bear market. For instance, Chen (1982) used a continuously changing time-varying parameter model to study beta nonstationarity across bull and bear market cycles.

As a consequence of these findings, the CAPM has been extended by using different risk measures for portfolio valuation, like the concept of drawdown measures. Initially, Markowitz (1959) proposed the mean-semi-variance approach to portfolio selection. Both standard deviation and lower semi-deviation are treated as general deviation measures that are not symmetric with respect to the rising and declining market trends of a random variable. Afterward, various non-symmetric risk measures have been proposed as an alternative to the standard beta, which is calculated over long periods to account for different market cycles. One of the proposals was the downside beta, which measures a stock's sensitivity to market downturns by focusing only on periods when the overall market's return is negative. This downside beta builds on the idea of downside risk. Jahankhani (1976) tested the risk and return relationships for a mean-variance and a mean-semivariance CAPM, but he did not find any improvement of the downside beta over the standard beta when predicting portfolios' returns. In contrast, Ang and Chen (2007) demonstrated that the usual downside beta has strong explanatory power for returns over the same period, but not for future returns. Even though Estrada (2002) showed that the downside beta explains the variation in the cross-section of asset returns in emerging markets, proposing the downside CAPM, or D-CAPM for short. He strongly supported the downside beta and the D-CAPM idea. Lettau, Maggiori, and Weber (2014) proved the explanatory power of the downside beta by the cross-section of currency returns. They showed that downside risk CAPM could rationalize the cross-section of equity, equity index options, commodity, sovereign bond, and currency returns, thus offering a unified risk view of the selected financial instruments. A new approach to portfolio optimization and risk reduction was proposed by Rockafellar and Uryasev (2000), who suggested minimizing the Conditional Value-at-Risk (CVaR) rather than the value-at-risk (VaR). This approach has gained popularity among investment companies, mutual funds, and international investors as a way to evaluate risk (Rockafellar and Uryasev, 2002, 2013).

Building on this, Rockafellar et al. (2006) extended classical CAPM by introducing non-symmetric risk measures known as Generalized Deviations. The classical portfolio theory goes beyond the traditional reliance on standard deviation. Rockafellar et al. (2006) stated that the CAPM equations are necessary optimality conditions for portfolio

optimization, where beta can be computed for CVaR and the lower semi-deviation (the square root of the semi-variance). Liu (2023) introduced a novel measure of downside beta, the ES-implied beta, based on expected shortfall, to improve the prediction of the cross-section of asset returns. The ES-implied beta explains stock returns similarly to the widely used downside beta, but it can be associated with high persistence. Finally, Krokhmal et al. (2011) provided a comprehensive review of these non-symmetric risk measures and the latest advances in decision-making under uncertainty. Their paper also introduced new formulas for estimating beta.

An important lesson from this discussion is that portfolio managers aim to construct portfolios that avoid drawdowns and minimize losses. Market drawdowns are commonly assessed using the widely recognized Maximum Drawdown, which measures the largest cumulative portfolio loss within a specified time interval (Daehwan, 2014). However, from an investor's perspective, the maximum drawdown is not an ideal risk measure as it reflects only one specific event on a price sample path.

To address this limitation, Goldberg and Mahmoud (2016) introduced the Conditional Expected Drawdown (CED), defined as the tail mean of the maximum drawdown distribution. Similarly, Chekhlov et al. (2004) proposed Conditional Drawdown at Risk (CDaR), which averages a specified percentage of the largest portfolio drawdowns over the whole investment horizon. CDaR has been used to identify systemic dependencies in the financial market and is conceptually similar to the CVaR, applied specifically to the drawdowns of cumulative portfolio returns. Ding and Uryasev (2022) employed CDaR regression to measure systemic risk contributions of financial institutions and for fund style classification. Allen et al. (2016) also analysed the downside risk metrics as a portfolio diversification strategy in a European market context by using CVaR optimisers and drawdown optimisers (MaxDD, AveDD, CDaR95, CDaRMin95).

Zabarankin et al. (2014) introduced the CAPM with drawdown beta, which provides the necessary optimization conditions in the form of CAPM equations. In this model, drawdown alpha and beta are defined analogously to the classical CAPM. The new formula for CDaR Beta was derived similarly to the standard beta, but it quantifies the relationship between market returns and individual asset returns exclusively during market drawdown periods. Both the drawdown beta and alpha were used to prioritize hedge fund strategies designed to mitigate market downturn risks. In summary, drawdown beta and standard beta have a clear distinction: drawdown beta focuses solely on asset performance during market drawdowns, effectively identifying hedging properties, whereas standard beta reflects the correlation between asset and market returns across all market conditions[4].

Broadly defined, a safe-haven asset is one that either retains or increases in value during periods of market turmoil or extreme financial stress. This concept has been extensively explored in academic literature, particularly in the context of financial crisis and portfolio risk management (Baur & Lucey, 2010; Baur et al., 2021). When describing the safe-haven behaviour, the main idea is to reduce portfolio risk in extreme market conditions, acting as a stabilizer when financial markets are under stress. Empirical identification of safe-haven assets remains contingent on various factors. The concept of drawdown risk measures as downside protection indicators is perfectly suited here, as the notion of drawdown beta has emerged as a complementary framework for understanding asset performance under adverse market conditions. A low or negative drawdown beta indicates that an asset either resists or inversely responds to market drawdowns, thereby providing protection during a crisis.

By refining the concept of drawdown beta, Ding and Uryasev (2022) introduced the Expected Regret of Drawdown (ERoD), a novel portfolio performance risk measure that computes the average drawdown exceeding a specified threshold. Essentially, the ERoD beta is constructed by dividing the average stock losses during market drawdowns that exceed the threshold by the corresponding average market drawdowns. This ERoD beta yields different results than the downside beta based on lower semi-deviation. While the ERoD beta is similar to CDaR beta, which averages a specified percentage of the largest drawdowns, it offers the advantage of explicitly quantifying the magnitude of the drawdowns. Although the CDaR and ERoD portfolio optimization problems are equivalent and yield the same optimal portfolios, they capture different drawdown characteristics. Empirical applications of ERoD and CDaR beta could enrich the existing literature on safe-haven assets by providing a quantitative metric for evaluating their effectiveness across different markets, by using the non-correlation definitions. Specifically, stocks that qualify as safe-haven assets should exhibit a low or negative drawdown beta. Both drawdown betas can be used effectively to identify safe-haven behaviour, especially when used together: CDaR Beta reveals how much an asset can lose in a worst-case drawdown. ERod Beta indicates whether an asset moves with or against the market during a crisis. This makes them valuable tools in risk-based asset selection and portfolio hedging.

4 See more about beta parameter and drawdown beta concept in Feder-Sempach E., (2024). Portfolio Management in Times of Elevated Risk. Safe-Haven and Hedge Assets in CAPM Setting, Journal of Finance and Financial Law.

METHODOLOGY

To calculate the ERoD and CDaR Betas, the European stock market was selected, represented by the STOXX Europe 600 index. It encompasses 600 large, mid, and small-cap companies across 17 European countries, denominated in both euros and their domestic currencies. It provides country and industry coverage, replicating almost 90% of the underlying investable market. This study uses daily and monthly closing prices of STOXX Europe 600 companies listed on the main European stock exchanges. All companies' quotations were converted into euros before calculating beta coefficient values, and we used the full time series with 428 stocks. The return frequency is daily for the drawdown beta (i.e., drawdown periods can vary in length) and monthly for the standard beta. According to the formulas below, the period of drawdown is selected, which means that drawdown betas are inherently conditional. They focus only on periods when the market is experiencing a drawdown, which are irregular intervals of stock returns. All stock data, including the STOXX Europe 600 index (price, net return in EUR), was collected between March 2004 and March 2024 from the LSEG Refinitiv EIKON database.

All companies were classified according to the Refinitiv Business Classifications (TRBC), the most comprehensive and up-to-date industry classification for analyzing companies and sectors across European markets[5]. This classification represents 10 main industries: Energy, Basic Materials, Consumer Cyclicals, Consumer Non-Cyclicals, Financials, Healthcare, Technology, Utilities, Real Estate, and Industrials.

This paper compares two drawdown betas – CDaR Beta and ERoD Beta – with the traditional beta. The CDaR Beta is defined as the average loss of a security over a percentage of the largest market drawdown periods, divided by the average market loss during those same periods. The ERoD beta is calculated as the average losses of a security during periods when the market experiences a drawdown exceeding a specified threshold ϵ, divided by the average market losses during those same periods (Ding & Uryasev, 2022). The formula for calculating the CdaR Beta and ERoD beta for security *i* is as follows:

$$\hat{\beta}^i_{CDaR,\alpha} = \frac{\sum_{t=1}^{T} q_t\left(w^i_{\tau(t)} - w^i_t\right)}{\sum_{t=1}^{T} q_t\left(w^M_{\tau(t)} - w^M_t\right)},$$

$$\hat{\beta}^i_{ERoD,\epsilon} = \frac{\frac{1}{T}\sum_{t=1}^{T} q^*_t\left(w^i_{\tau(t)} - w^i_t\right)}{\frac{1}{T}\sum_{t=1}^{T} q^*_t\left(w^M_{\tau(t)} - w^M_t\right)},$$

where:
i – index of security $i = 1, \ldots I$,
t – time, $t = 1, \ldots, T$,
w^i_t – compounded cumulative return of asset i at time t,
w^M_t – compounded cumulative return of the market portfolio at time t,
$d^M_t = w^M_{\tau(t)} - w^M_t$ drawdowns of the market portfolio,
q_t –indicator function, which is equal to $1/\left((1-\alpha)T\right)$ if d^M_t is one of the largest $(1-\alpha)T$ drawdowns $d^M_1, d^M_2, \ldots, d^M_T$ of the market portfolio, and 0 otherwise,
$q^*_t = 1(d^M_t \geq \epsilon)$ – indicator function, which is equal to 1 for $d^M_t \geq \epsilon$, and 0 otherwise.
$\tau(t) = \max\{k | 1 \leq k \leq t, w^M_k = \max_{1 \leq l \leq t} w^M_l\}$.

For the CdaR Beta, the parameter $\alpha \in [0,1]$ controls the percentage of the largest drawdowns included in the calculation. Specifically, $\alpha = 0$ includes all market drawdowns, while $\alpha = 0$ considers only the maximum (i.e., the single largest) market drawdown. For example, the $CdaR_{0.9}$ beta accounts for the largest 10% of drawdowns in the market portfolio.

For the ERoD beta, the parameter $\epsilon \in (0,+\infty)$ determines the magnitude of the market drawdowns to be considered. Ding and Uryasev (2022) proposed a threshold of 0 + with $\epsilon = 10^{-6}$ to account for all non-zero drawdowns. In the empirical part of the study, we calculated drawdown betas with the following parameters: the CdaR beta with $\alpha = 0.9$ ($CdaR_{0.9}$) and

5 Our sample does not include any companies from Luxembourg.

the ERoD beta with a threshold of $\epsilon = 10^{-6}$ ($EroD_{0+}$), both based on daily returns[6]. The standard beta was calculated using monthly returns based on Fama and French's (2004) formula:

$$\hat{\beta}^i_{standard} = \frac{cov(r^i_t, r^M_t)}{var(r^M_t)},$$

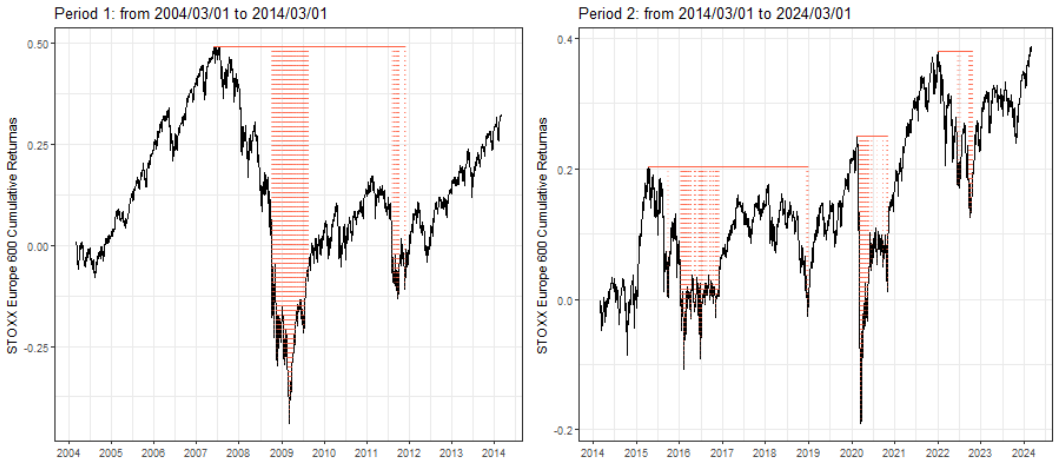
where:
i – index of security $i = 1, \ldots I$,
t – time, $t = 1, \ldots, T$,
r^i_t – return of asset i at time t,
r^M_t – return of the market portfolio at time t.

It is important to note that the CdaR Beta and the ERoD Beta capture different drawdown characteristics. The CdaR Beta focuses on unique market conditions, such as the GFC and the COVID-19 crisis, which are unlikely to recur. In contrast, the EroD Beta accounts for all nonzero drawdowns in the STOXX Europe 600. Thus, the EroD Beta should be treated as an alternative to the CdaR Beta.

RESULTS AND DISCUSSION

We begin by comparing the ERoD Beta, the CDaR Beta, and the standard beta for European companies in two subperiods. Comparing drawdown betas with the standard beta demonstrates that, although the traditional beta is more stable, it provides little additional information during periods of market distress. In contrast, drawdown betas are more sensitive to market distress during unexpected events. Their tendency to take on negative values during such times may indicate potential safe-haven characteristics (cf. Bogołębska et al. 2024), offering greater informative power to investors focused on European markets.

Below, we compare the values of betas across two distinct periods. The first period reflects the impact of the GFC and the Eurozone crisis, while the second period encompasses several major events, including Donald Trump's first election, the Brexit crisis, the COVID-19 economic crisis, and Russia's invasion of Ukraine (Figure 1):



Note: The 10% largest drawdowns are highlighted in red. Period 1: 2004 – 2014, dominated by the GFC; Period 2: 2014 – 2024, dominated by the COVID-19 economic crisis.

Figure 1. STOXX Europe 600 in Periods 1 and 2
Source: Own elaboration based on LSEG Refinitiv EIKON data.

6 The drawdown betas are not computed on daily intervals, but the return time series is daily.

The analysis is based on cross-country and industry patterns, as well as on deciles of European companies ranked by the means of year-end market capitalization[7]. The first decile represents the 10% of companies with the smallest market cap, while the tenth decile covers the 10% biggest companies. Table 1 presents average beta values by country; Table 2 shows average beta values by industry; and Table 3 displays average beta values by market capitalization deciles for Periods 1 and 2. The corresponding results based on medians are reported in Tables A1–A3 in the appendix.

Table 1. Average beta values by country in Periods 1 and 2

Country of exchange	Period 1			Period 2			N
	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	
Austria	1.456	1.160	1.212	0.002	0.914	1.068	6
Belgium	1.059	0.936	0.879	-0.252	0.383	0.825	12
Denmark	1.068	1.091	0.978	0.258	0.699	1.095	20
Finland	1.156	1.083	1.034	0.398	0.719	0.929	12
France	1.040	0.975	1.048	0.154	0.682	1.068	58
Germany	0.783	0.899	1.031	0.464	0.738	1.089	47
Ireland; Republic of	2.966	1.229	1.028	1.479	0.820	0.951	6
Italy	1.711	1.102	0.936	1.643	1.140	1.191	25
Netherlands	0.930	0.987	1.074	0.787	0.998	1.114	18
Norway	0.996	0.943	1.102	-0.265	0.386	0.749	11
Poland	0.551	1.074	1.079	0.407	1.125	1.040	6
Portugal	1.277	0.702	0.727	2.881	1.142	0.930	3
Spain	1.082	0.832	0.928	0.962	0.935	0.948	17
Sweden	0.766	1.038	1.096	0.279	0.675	1.087	43
Switzerland	0.431	0.705	0.675	-0.121	0.497	0.771	44
United Kingdom	1.179	1.077	0.953	1.118	0.978	1.100	100
Total	1.022	0.983	0.973	0.572	0.788	1.033	428

Source: Own elaboration based on LSEG Refinitiv EIKON data.

Table 2. Average beta values by industry in Periods 1 and 2

Sector	Period 1			Period 2			N
	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	
Basic Materials	0.750	0.952	1.141	0.424	0.881	1.069	39
Consumer Cyclicals	1.087	1.100	1.027	0.914	0.979	1.160	60
Consumer Non-Cyclicals	0.493	0.760	0.649	0.147	0.359	0.753	38
Energy	0.797	0.806	1.053	1.166	0.916	1.068	17
Financials	1.863	1.223	1.255	1.768	1.291	1.206	80
Healthcare	0.530	0.717	0.596	-0.551	0.170	0.833	34
Industrials	0.814	0.971	1.022	0.179	0.717	1.084	75
Real Estate	1.165	1.072	0.752	1.079	0.936	0.884	19
Technology	0.807	0.922	0.905	-0.561	0.392	0.977	46
Utilities	1.165	0.865	0.733	0.860	0.823	0.808	20
Total	1.022	0.983	0.973	0.572	0.788	1.033	428

Source: Own elaboration based on LSEG Refinitiv EIKON data.

7 First, end-of-year market capitalizations were selected; then, their means were calculated; finally, companies were assigned to deciles based on these means.

Table 3. Average beta values in Periods 1 and 2, relative to deciles of company value

Deciles of company value	Period 1			Period 2			N
	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	
1st decile	0.858	0.922	0.726	-0.054	0.685	0.949	43
2nd decile	1.227	1.107	0.901	0.178	0.610	1.066	43
3rd decile	1.011	0.950	0.928	0.599	0.704	0.986	43
4th decile	1.309	1.121	1.034	0.610	0.791	1.076	42
5th decile	1.124	0.987	0.935	0.417	0.764	1.039	43
6th decile	0.979	0.912	0.916	0.491	0.695	0.978	43
7th decile	1.009	0.996	1.029	0.606	0.862	1.052	42
8th decile	1.207	1.054	1.095	0.999	0.987	1.072	43
9th decile	0.844	0.902	1.111	0.726	0.869	1.073	43
10th decile	0.658	0.887	1.061	1.150	0.915	1.039	43
Total	1.022	0.983	0.973	0.572	0.788	1.033	428

Source: Own elaboration based on LSEG Refinitiv EIKON data.

According to Table 1, CDaR and standard beta are always positive on average, making it impossible to identify European companies that can hedge a portfolio by moving in the opposite direction from the STOXX Europe 600 index. However, for the ERoD beta, the average value for Belgian, Norwegian, and Swiss companies is negative in Period 2, which covers the COVID-19 pandemic. Norway and Switzerland, which are not members of the European Union, are at a different stage of the business cycle than EU countries[8]. One explanation is that changes in the external environment tend to impact Norwegian and Swiss companies differently from those in the EU. Although the global economy is increasingly interdependent during a crisis due to spillover effects, some Norwegian and Swiss companies remain unaffected and could be used to hedge European portfolios (Reskiyah, 2023). Belgian companies were also relatively resilient to the recent crisis despite Belgium being an EU and eurozone member. It is also worth noting the exceptionally high ERoD Beta values for Irish companies – 2.966 in Period 1 and 1.479 in Period 2, along with similarly elevated values for Italian and Portuguese issuers. This may indicate that Irish, Italian, and Portuguese companies were highly sensitive to the crisis, or they may simply reflect an overweight of financial firms or the dominance of small-cap companies, rather than a genuine country-specific effect.

As shown in Table 2, the situation remains largely the same: the CDaR and standard beta values are positive on average. However, the ERoD beta shows negative values for the Healthcare and Technology industries in Period 2, which coincided with the COVID-19 pandemic crisis. These results show that Healthcare and Technology companies may have experienced accelerated returns due to heightened demand for medical and digital services caused by the COVID-19 crisis in Europe. The pandemic accelerated the digital transformation of many services and positively impacted companies in the Healthcare and Technology industries. Those industries became strategically important during the pandemic due to vaccine development, increased demand for medical products, and global lockdowns and remote work. Technology and healthcare stocks acted as resilient during the pandemic, similarly to gold, which is traditionally seen as a safe-haven asset (Baur & McDermott, 2010). Investors buy gold when uncertainty is high (e.g., during geopolitical turmoil, pandemics, or financial crises) because gold is viewed as a store of value not tied to the performance of economies. The pandemic created a unique situation where both gold and certain stocks performed well.

Gained investor interest as defensive stocks — demand for healthcare tends to remain stable even during economic downturns. According to Nassar et al. (2023), portfolio analysis indicates that investors should include the Healthcare and Technology sectors in their equity portfolios to reduce investment risk and protect expected returns during a pandemic.

Conversely, the highest beta values are observed in the Financial industry, which appears more unstable relative to the market than other industries, not only during periods of elevated risk but also under normal market conditions.

As shown in Table 3, similar results are observed in the ERoD Beta values for smaller, listed European companies. Specifically, for firms in the first decile based on market capitalization, the average ERoD Beta value is negative in Period

8 Norway and Switzerland are ranked as the most developed and richest countries from the global perspective while Norway is a member of the European Economic Area whereas Switzerland can participate and access the European single market through a set of bilateral agreements with the EU.

2 for the smallest 10% of companies by market capitalization. Generally, ERoD and CDaR Beta values are much smaller in Period 2, with nearly all values below 1.

In accordance with the study results, the ERoD and CDaR Beta values are, in some cases, lower than standard beta values, which are generally close to 1. Furthermore, during Period 1 (which covered the GFC), the ERoD and CDaR Betas were much higher than in Period 2 (which covered COVID-19), suggesting that European companies reacted differently to the distinct nature of each crisis. The ERoD Beta appears to deliver the best results because it captures the hedging attributes of selected European companies across countries, industries, and market capitalization, thereby improving portfolio efficiency. The average CDaR and standard beta contribution values remain positive across both periods despite the contrasting market shocks.

Tables 4–6 present the correlation coefficients for ERoD, CDaR, and standard betas between Period 1 and Period 2 across countries, industries, and company market capitalization, respectively. The negative correlation coefficients between these two periods show that European companies reacted differently to the distinct market drawdowns of the GFC and COVID-19.

Table 4. Beta correlations (Pearson correlation coefficients with 95% confidence intervals in parentheses) between Periods 1 and 2 in individual countries

Country of Exchange	Correlation			N
	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	
Austria	-0.145 (-0.856; 0.755)	0.164 (-0.747; 0.861)	0.704 (-0.250; 0.965)	6
Belgium	0.309 (-0.322; 0.750)	0.760 (0.331; 0.929)	0.798 (0.414; 0.941)	12
Denmark	-0.231 (-0.611; 0.236)	0.220 (-0.246; 0.604)	0.864 (0.682; 0.945)	20
Finland	0.117 (-0.490; 0.647)	0.021 (-0.559; 0.588)	0.900 (0.676; 0.972)	12
France	0.241 (-0.019; 0.470)	0.345 (0.095; 0.554)	0.735 (0.588; 0.835)	58
Germany	0.185 (-0.108; 0.448)	-0.023 (-0.308; 0.266)	0.787 (0.647; 0.876)	47
Ireland; Republic of	0.797 (-0.040; 0.977)	0.666 (-0.316; 0.959)	0.855 (0.142; 0.984)	6
Italy	0.629 (0.311; 0.820)	0.610 (0.283; 0.810)	0.771 (0.541; 0.894)	25
Netherlands	0.261 (-0.235; 0.649)	0.586 (0.164; 0.827)	0.809 (0.549; 0.926)	18
Norway	-0.068 (-0.642; 0.555)	-0.197 (-0.713; 0.457)	0.776 (0.330; 0.939)	11
Poland	0.840 (0.090; 0.982)	0.861 (0.165; 0.985)	0.247 (-0.706; 0.882)	6
Portugal	0.965	0.981	0.996	3
Spain	0.024 (-0.462; 0.499)	0.246 (-0.266; 0.650)	0.751 (0.422; 0.905)	17
Sweden	-0.093 (-0.382; 0.214)	-0.011 (-0.311; 0.290)	0.610 (0.380; 0.770)	43
Switzerland	-0.074 (-0.362; 0.228)	0.223 (-0.079; 0.488)	0.832 (0.710; 0.905)	44
United Kingdom	0.294 (0.104; 0.464)	0.380 (0.198; 0.536)	0.741 (0.637; 0.818)	100
Total	0.311 (0.223; 0.394)	0.300 (0.211; 0.384)	0.722 (0.673; 0.764)	428

Note: No confidence interval is reported for Portugal due to an insufficient number of observations.
Source: Own elaboration based on LSEG Refinitiv EIKON data.

Table 5. Beta correlations (Pearson correlation coefficients with 95% confidence intervals in parentheses) between Periods 1 and 2 in individual industries

Sector	Correlation			N
	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	
Basic Materials	-0.069 (-0.376; 0.252)	0.094 (-0.228; 0.398)	0.867 (0.759; 0.929)	39
Consumer Cyclicals	0.221 (-0.035; 0.450)	0.298 (0.047; 0.513)	0.639 (0.460; 0.768)	60
Consumer Non-Cyclicals	0.326 (0.007; 0.584)	0.380 (0.069; 0.624)	0.390 (0.080; 0.631)	38
Energy	0.244 (-0.268; 0.649)	0.550 (0.095; 0.815)	0.562 (0.111; 0.821)	17
Financials	0.479 (0.290; 0.632)	0.486 (0.299; 0.638)	0.580 (0.413; 0.710)	80
Healthcare	-0.468 (-0.696; -0.154)	-0.399 (-0.649; -0.070)	0.577 (0.296; 0.766)	34
Industrials	-0.013 (-0.240; 0.214)	0.064 (-0.165; 0.287)	0.614 (0.450; 0.738)	75
Real Estate	0.260 (-0.220; 0.639)	0.463 (0.012; 0.758)	0.809 (0.560; 0.924)	19
Technology	0.069 (-0.226; 0.352)	0.037 (-0.256; 0.324)	0.524 (0.276; 0.707)	46
Utilities	-0.247 (-0.622; 0.219)	0.028 (-0.420; 0.465)	0.595 (0.207; 0.821)	20
Total	0.311 (0.223; 0.394)	0.300 (0.211; 0.384)	0.722 (0.673; 0.764)	428

Note: No confidence interval is reported for Portugal due to an insufficient number of observations.
Source: Own elaboration based on LSEG Refinitiv EIKON data.

Table 6. Beta correlations (Pearson correlation coefficients with 95% confidence intervals in parentheses) between periods 1 and 2 in individual deciles of the company value

Deciles of company value	Correlation			N
	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	
1st decile	0.072 (-0.233; 0.365)	0.275 (-0.027; 0.532)	0.729 (0.549; 0.845)	43
2nd decile	0.264 (-0.039; 0.523)	0.351 (0.057; 0.589)	0.625 (0.400; 0.779)	43
3rd decile	0.510 (0.248; 0.703)	0.350 (0.055; 0.588)	0.752 (0.584; 0.859)	43
4th decile	0.303 (-0.001; 0.556)	0.403 (0.113; 0.630)	0.664 (0.451; 0.805)	42
5th decile	0.110 (-0.197; 0.397)	0.065 (-0.240; 0.359)	0.732 (0.554; 0.847)	43
6th decile	0.617 (0.389; 0.774)	0.596 (0.360; 0.760)	0.721 (0.537; 0.839)	43
7th decile	0.286 (-0.020; 0.543)	0.296 (-0.009; 0.550)	0.835 (0.711; 0.908)	42
8th decile	0.566 (0.320; 0.741)	0.541 (0.287; 0.724)	0.897 (0.816; 0.943)	43
9th decile	-0.196 (-0.469; 0.111)	0.026 (-0.276; 0.324)	0.610 (0.379; 0.769)	43
10th decile	0.515 (0.254; 0.706)	0.331 (0.035; 0.575)	0.643 (0.425; 0.791)	43
Total	0.311 (0.223; 0.394)	0.300 (0.211; 0.384)	0.722 (0.673; 0.764)	428

Source: Own elaboration based on LSEG Refinitiv EIKON data.

Tables 4–6 show the Pearson correlation coefficients with 95% confidence intervals in parentheses between the two periods. Tables A4–A6 in the appendix show the corresponding Spearman correlations. All correlation coefficients calculated for the standard beta between the two periods are, on average, positive, indicating that the standard beta cannot capture variations in stock behavior across unrelated market crises. In contrast, the correlation coefficients for the ERoD and CDaR Betas are negative, suggesting that stock performance differed by crisis type.

Notably, Austrian, Danish, Norwegian, Swedish, and Swiss companies displayed distinct behavior in terms of the ERoD Beta. A similar pattern is observed across industries such as Basic Materials, Healthcare, Industrials, and Utilities, as well as among companies with higher market capitalizations. The CDaR Beta showed fewer negative correlation coefficients, primarily among companies from Germany, Norway, and Sweden, and in the Healthcare sector. Consistent with earlier findings, this supports the inclusion of Healthcare companies in equity portfolios to reduce investment risk during periods such as the COVID-19 crisis.

According to the numerical results, there is no relationship between a company's market capitalization and its beta during the drawdown periods of the GFC and COVID-19. This finding contradicts the results of Ding and Uryasev (2022), who reported that larger companies tend to exhibit stronger correlations in beta coefficients across crisis periods.

According to the classic CAPM interpretation, beta measures the sensitivity of a stock's return to variation in the market index return (Fama & French, 2004). As such, a positive beta indicates that the asset moves in the same direction as the market, while a negative beta would indicate the opposite. In the context of portfolio optimization, a contingency analysis of positive and negative ERoD and CDaR Beta values is proposed. Tables 7 and 8 present the number of companies with negative and positive ERoD and CDaR Betas across the two periods. In the case of the standard beta, no company has a negative beta in any period.

Table 7. Contingency table of positive and negative $ERoD_{0+}$ beta values in Periods 1 and 2

$ERoD_{0+}$ Beta	**Period 2**		
Period 1	Negative (-)	Positive (+)	Total
Negative (-)	21	30	51
Positive (+)	129	248	377
Total:	150	278	428

Source: Own elaboration based on LSEG Refinitiv EIKON data.

Table 8. Contingency table of positive and negative $CDaR_{0.9}$ beta values in Periods 1 and 2

$CDaR_{0.9}$ Beta	**Period 2**		
Period 1	Negative (-)	Positive (+)	Total
Negative (-)	2	5	7
Positive (+)	58	363	421
Total:	60	368	428

Source: Own elaboration based on LSEG Refinitiv EIKON data.

The contingency tables show the number of stocks with negative and positive ERoD and CDaR Beta values in Periods 1 and 2. No negative standard beta values – that would indicate an inverse relationship with the market – were observed. According to Table 7, 21 companies have a negative ERoD Beta in Periods 1 and 2, while 30 companies have a negative ERoD Beta in Period 1 but a positive one in Period 2. Conversely, 129 companies have an ERoD Beta that is positive in Period 1 but negative in Period 2. Meanwhile, 248 companies maintain a positive ERoD Beta across both periods.

Table 8 reveals that only two companies exhibit a negative CDaR Beta in both periods. Five companies have a negative CDaR Beta in Period 1 and a positive value in Period 2. In contrast, 58 companies have a CDaR Beta that is positive in Period 1 but negative in Period 2. The vast majority – 363 companies – have a positive CDaR Beta across both periods. These results indicate that the majority of European stocks moved in the same direction as the market, as indicated by the ERoD and CDaR Beta evaluations. However, these two new beta measures can potentially identify stocks that can rise during market distress and help investors diversify their portfolios.

In line with Ding and Uryasev's (2022) research on the American market, this study finds that ERoD and CDaR Betas applied to the European market are better at identifying stocks that can hedge portfolios and might serve as

indicators of downside protection measures in effective risk management strategies. In classical financial theory, safe-haven stocks tend to maintain or even gain value during market downturns. Consistently low or negative ERoD and CDaR Beta during crises could be considered safe from drawdown risk and extremely useful for strategies concerned with capital preservation. Surprisingly, there is no relationship between the market capitalization of the analyzed European companies and the beta in the two main drawdown periods of the GFC and COVID-19, which contrasts with the results observed in the US market. Larger stocks tend to have more stable betas on average and more stable downside exposure, due to greater diversification, better financing, more elastic access to capital, and, generally, a more stable business as a whole (Ding & Uryasev, 2022).

CONCLUSION

The drawdown betas extend beyond the traditional beta, which measures sensitivity to market movements (market exposure coefficient), and specifically focuses on an asset's performance during market downturns (drawdown exposure coefficient). In essence, drawdown betas capture stock performance when the market is in a drawdown. The study revealed that drawdown betas are more sensitive to market downturns than the standard beta. While the standard beta is always positive for all firms (the minimum value observed in the sample is 0.078), some European stocks exhibit negative drawdown betas, indicating that these stocks generate positive returns when the market is in drawdown and can work as diversifying assets. These findings support the hypothesis that drawdown betas have fundamentally different characteristics from traditional betas and can be negative when the standard beta is positive. Relying solely on the standard beta might cause investors to miss the safe-haven effect and fail to adequately hedge their portfolios in the event of a crisis. The drawdown betas provide a valuable tool for portfolio construction.

Our empirical analysis across European countries, industries and market capitalization deciles indicates that standard beta are always positive and cannot identify companies that could hedge or diversify portfolios. Only the ERoD and CDaR Beta display negative values. The industry analysis showed that companies in the Healthcare and Technology sectors may have safe-haven attributes during market drawdowns, as reflected by their ERoD and CDaR Beta values, which are negative or below 1. This outcome is consistent with the context of the COVID-19 pandemic crisis, which disproportionately benefited Healthcare companies during the market collapse, making them similar to gold that serves its classic role as a safe-haven asset.

The correlation coefficients between Periods 1 and 2 of the ERoD, CDaR, and standard betas across countries, industries, and market capitalization values once again showed that the standard beta cannot capture differences in stock behavior across unrelated market downturns. Industries such as Basic Materials, Healthcare, Industrials, and Utilities exhibited different ERoD Beta behaviors during the GFC and COVID-19 crisis. Moreover, no consistent relationship was found between market value and beta values during the GFC drawdown period and the COVID-19 period, in contrast to the results of Ding and Uryasev (2022).

In line with the contingency table, most European stocks moved in the same direction as the market, but we hope that new beta measures can identify stocks that may help diversify portfolios. The ERoD and CDaR Beta values tend to be lower than standard beta values, which are mostly close to 1. The study revealed that drawdown betas, especially ERoD, are more sensitive to market drawdowns and hold greater informational value than the standard beta. Our findings shed light on a new significant risk measure: the drawdown beta.

A more precisely estimated investment risk is highly desirable and effective in financial market analysis, offering better returns on investments. The findings of this research could help investors quantify investment risk across European companies when unprecedented events, such as the GFC or the COVID-19 crisis, present challenges for both investors and financial analysts. The drawdown beta offers valuable insights for constructing more resilient portfolios, effectively managing risk, and evaluating performance amidst market fluctuations. This study shifts the focus from static correlation measures to dynamic, downside-sensitive risk metrics, aligning theoretical definitions with real-world investor concerns about capital preservation during drawdowns. This gives rise to a legitimate hope that the drawdown beta can be applied in other highly developed European markets or, perhaps, in emerging economies.

This study, however, is not without its limitations. Thus, future research should incorporate the two-factor International Capital Asset Pricing Model (ICAPM) to account for exchange-rate risk premiums across European countries. We also identified the weaknesses of the dataset. We have included 428 companies, but they should be taken with caution due to the survivorship bias, as only surviving companies were included in the analysis.

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Appendix

Table A1. Median beta values by country in Periods 1 and 2

Country of exchange	Period 1			Period 2			N
	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	
Austria	1.378	1.234	1.180	-0.181	0.912	1.160	6
Belgium	0.778	0.850	0.796	-0.169	0.357	0.804	12
Denmark	1.226	1.194	0.986	0.423	0.882	1.144	20
Finland	0.955	1.091	0.965	0.298	0.802	0.936	12
France	0.983	0.966	1.032	0.453	0.821	1.035	58
Germany	0.707	0.881	1.030	0.353	0.805	1.071	47
Ireland; Republic of	1.838	1.264	0.987	0.262	0.348	1.023	6
Italy	1.675	1.110	0.911	0.795	1.265	1.203	25
Netherlands	0.836	1.030	1.020	0.384	0.991	1.183	18
Norway	1.085	1.158	1.100	-0.306	0.494	0.773	11
Poland	0.544	1.090	1.148	1.547	1.727	1.062	6
Portugal	1.019	0.774	0.704	1.102	0.930	0.804	3
Spain	1.276	0.919	0.888	0.679	0.775	0.985	17
Sweden	0.580	0.988	1.031	0.510	0.933	1.090	43
Switzerland	0.311	0.641	0.608	0.099	0.608	0.783	44
United Kingdom	0.899	1.017	0.931	1.254	1.043	1.077	100
Total	0.842	0.996	0.955	0.509	0.868	1.033	428

Source: Own elaboration based on LSEG Refinitiv EIKON data.

Table A2. Median beta values by industry in Periods 1 and 2

Sector	Period 1			Period 2			N
	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	
Basic Materials	0.600	1.027	1.165	0.496	0.926	1.042	39
Consumer Cyclicals	0.848	1.112	1.068	1.157	1.115	1.170	60
Consumer Non-Cyclicals	0.410	0.806	0.633	0.017	0.335	0.734	38
Energy	0.708	0.781	1.043	1.022	0.844	1.151	17
Financials	1.580	1.301	1.254	1.307	1.214	1.181	80
Healthcare	0.542	0.679	0.583	0.156	0.440	0.832	34
Industrials	0.891	0.996	1.047	0.082	0.780	1.104	75
Real Estate	1.337	1.248	0.869	0.760	0.891	0.939	19

Sector	Period 1			Period 2			N
	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	
Technology	0.705	0.960	0.907	-0.472	0.461	0.985	46
Utilities	1.087	0.857	0.721	1.030	0.772	0.803	20
Total	0.842	0.996	0.955	0.509	0.868	1.033	428

Source: Own elaboration based on LSEG Refinitiv EIKON data.

Table A3. Median beta values in Periods 1 and 2, relative to deciles of company value

Deciles of company value	Period 1			Period 2			N
	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	
1st decile	0.850	0.979	0.692	0.170	0.762	0.957	43
2nd decile	1.237	1.142	0.948	0.008	0.716	1.057	43
3rd decile	0.707	0.956	0.901	0.407	0.696	0.946	43
4th decile	1.154	1.090	1.013	0.691	0.929	1.132	42
5th decile	1.015	0.915	0.908	0.243	0.872	0.988	43
6th decile	0.538	0.856	0.870	0.428	0.607	0.984	43
7th decile	1.068	0.995	0.952	0.904	0.981	1.058	42
8th decile	1.045	1.026	1.031	0.873	0.998	1.053	43
9th decile	0.737	0.927	1.065	0.800	0.953	1.090	43
10th decile	0.424	0.895	1.060	0.798	0.941	1.104	43
Total	0.842	0.996	0.955	0.509	0.868	1.033	428

Source: Own elaboration based on LSEG Refinitiv EIKON data.

Table A4. Beta correlations (Spearman correlation coefficient) between Periods 1 and 2 in individual countries

Country of exchange	Correlation			N
	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	
Austria	-0.086	-0.029	0.714	6
Belgium	0.420	0.210	0.783	12
Denmark	-0.205	0.238	0.879	20
Finland	-0.182	-0.182	0.846	12
France	0.369	0.380	0.769	58
Germany	0.179	0.104	0.787	47
Ireland; Republic of	0.600	0.543	0.771	6
Italy	0.512	0.556	0.744	25
Netherlands	0.399	0.575	0.849	18
Norway	-0.127	-0.145	0.782	11
Poland	0.829	0.771	0.371	6
Portugal	1.000	1.000	1.000	3
Spain	-0.140	0.346	0.718	17
Sweden	-0.072	-0.068	0.592	43
Switzerland	-0.118	0.237	0.818	44
United Kingdom	0.305	0.436	0.776	100
Total	0.219	0.358	0.721	428

Source: Own elaboration based on LSEG Refinitiv EIKON data.

Table A5. Beta correlations (Spearman correlation coefficient) between Periods 1 and 2 in individual industries

Sector	Correlation			N
	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	
Basic Materials	-0.051	0.220	0.857	39
Consumer Cyclicals	0.239	0.328	0.684	60
Consumer Non-Cyclicals	0.100	0.326	0.486	38
Energy	0.181	0.525	0.520	17
Financials	0.512	0.478	0.571	80
Healthcare	-0.500	-0.329	0.485	34
Industrials	-0.010	0.093	0.611	75
Real Estate	0.140	0.560	0.653	19
Technology	0.018	0.113	0.508	46
Utilities	-0.343	-0.077	0.573	20
Total	0.219	0.358	0.721	428

Source: Own elaboration based on LSEG Refinitiv EIKON data.

Table A6. Beta correlations (Spearman correlation coefficient) between periods 1 and 2 in individual deciles of the company value.

Deciles of company value	Correlation			N
	$ERoD_{0+}$	$CDaR_{0.9}$	Standard	
1st decile	0.040	0.183	0.730	43
2nd decile	0.317	0.422	0.642	43
3rd decile	0.387	0.518	0.762	43
4th decile	0.232	0.406	0.622	42
5th decile	0.125	0.141	0.742	43
6th decile	0.282	0.600	0.689	43
7th decile	0.237	0.384	0.839	42
8th decile	0.545	0.599	0.876	43
9th decile	-0.137	0.099	0.681	43
10th decile	0.143	0.314	0.612	43
Total	0.219	0.358	0.721	428

Source: Own elaboration based on LSEG Refinitiv EIKON data.

Biographical notes

Ewa Feder-Sempach serves as Assistant Professor at the Faculty of Economics and Sociology, University of Lodz, Poland. Graduated from the University of Lodz, receiving master's degrees in International Economics (2004) and English Philology (2020). She received a Ph.D. in Economics from University of Lodz in 2010. She has been a visiting researcher at the Bayes Business School in London (UK), the University of Illinois Chicago (USA), and the University of Liverpool Management School (UK). In 2025, she completed the Advanced Risk and Portfolio Management (ARPM) Quant Bootcamp Certificate. Her scientific research includes international investments and portfolio analysis. She is also the author and co-author of several publications, more than forty articles, monographs, and chapters devoted to international financial markets, investment theory, and portfolio systematic risk.

Piotr Szczepocki serves as Assistant Professor at the Faculty of Economics and Sociology, University of Lodz, Poland. Graduated from the University of Lodz and Lodz University of Technology, receiving master's degrees in Econometrics (2010) and Mathematics (2012). He received a Ph.D. degree in Economics from Lodz University in 2019. In 2025, he completed the Advanced Risk and Portfolio Management (ARPM) Quant Bootcamp Certificate. His scientific research includes stochastic variability models and Monte Carlo experiments.

Stan Uryasev received his M.S. in Applied Mathematics from the Moscow Institute of Physics and Technology (MIPT), Russia, in 1979 and his Ph.D. in Applied Mathematics from the Glushkov Institute of Cybernetics, Kyiv, Ukraine, in 1983. From 1979 to 1987, he held a research position at the Glushkov Institute. From 1988 to 1992, he was a Research Scholar at the International Institute for Applied System Analysis in Laxenburg, Austria. From 1992 to 1998, he held the Scientist position at the Risk and Reliability Group at Brookhaven National Laboratory, Upton, NY. From 1998 to 2019, he was the George and Rolande Willis Endowed Professor at the University of Florida and the director of the Risk Management and Financial Engineering Lab. His research and teaching interests include quantitative finance, risk management, stochastic optimization, machine learning, and military operations research.

Author contributions statement

Ewa Feder-Sempach: Conceptualization, Data Curation, Writing – Original Draft Preparation, Writing – Review & Editing. **Piotr Szczepocki**: Conceptualization, Formal Analysis, Methodology, Writing – Original Draft Preparation. **Stan Uryasev**: Conceptualization, Methodology, Supervision.

Conflicts of interest

The authors declare no competing interests.

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Data availability

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