

A new method of DCF valuation

Marek Capiński

Nowy Sacz School of Business – NLU

Introduction.

Among many valuation methods the one based on computing the present value of the expected future cash flows is indisputably regarded as the best. The question is what kind of cash flow we should consider and what is the discount rate. The main goal is to estimate the market value of equity; therefore one should take into account the cash flow for the shareholders and use the cost of equity as the rate. This approach is called the direct method (see [BS]). The indirect method is based on free cash flows discounted by the weighted average cost of capital and then subtracting the value of debt, which, if the credit risk is ignored, is close to the book value and so is easily available.

Both methods are exposed to the following fundamental problem, ignored by most authors. The costs of capital (both cost of equity and WACC) depend on the financial structure. This in turn depends on the value of equity, which needs the cost of capital as an input to be computed. This loop is usually solved by assuming that the capital structure is constant in time, equal to some optimal, target structure, presumed known. Optimality is related of course to maximizing the equity value and so this assumption cannot be formulated in separation from the valuation process. The constant structure is under pressure from volatile profits thus variable equity value. The typical remedy here is the requirement that the level of debt be adjusted continuously to keep the structure constant. This is highly impractical and in this paper we allow the capital structure to vary.

The analysis is performed within the usual multi-period setup, where for simplicity we assume the length of one time step to be a year. We restrict our attention to a finite planning horizon where the final (residual) value is obtained by discounting a perpetuity (or growing perpetuity) with the assumption of constant structure imposed for the infinite tail of cash flows that follow the horizon date. There is little need to be bothered by the unrealistic nature of this assumption since we are dealing with the period which goes far (infinitely far) into the future and we are forced to make guesses in place of proper forecasting anyway.

Given this residual value (it can be zero if one expects the company to cease to exist) we move backwards in time, step by step, arriving eventually at the recent moment. Therefore, what we need is to compute the value at time t if the value together with the cash flows at time $t + 1$ is known. We follow the direct method focusing on the cash flow to equity, equity value, and cost of equity.

The main theoretical problem is to provide the way of estimating the cost of equity. The common tool here is CAPM, which takes into account the systematic (market) risk only assuming that the specific risk should be diversified by an investor. However, quite often, especially in small business, the owner cannot diversify in the financial markets since all her funds are tied in her business so the exposure to the specific risk cannot be ignored.

The main feature of the approach adopted here is the assumption that the required return (cost of capital) is equal to the expected return, which is determined by the expected cash flows. In other words, we assume that some equilibrium is reached. The argument behind this assumption is this: expected return higher than required leads to positive NPV but this opportunity, if noticed by the competition, should quickly cease to exist as a result of declining prices after supply is increased. If the expected return is below the required we do not enter the business at all, and so in a competitive market these two rates should equal.

Our approach is a refinement of the method proposed by P. Fernandez in his recent book [F1] and a series of articles. He describes an approach to valuation of companies with an emphasis on a correct estimation of the present value of tax shield. The main principle he applies is: the present value of the tax shield is the difference between the value of a (hypothetical) company financed entirely by equity (unlevered, hence the subscript u), and the value of the company with partial debt financing:

$$PVTS = V_u - V$$

with

$$V = D + E .$$

The questions to answer are:

1. The choice of the invariants: the quantities that are independent of the financial structure.
2. The choice of the discount rate k_E to find E as the present value of cash flow to equity ECF .
3. The choice of the discount rate k_u to find V_u as the present value of the free cash flow FCF .
4. The relation between k_E , k_u , k_D and the financial structure.

The answers should give a method applicable to a general situation where the life span, the level of earnings and the level of debt are arbitrary. For this purpose it is sufficient to analyze carefully a one period case properly incorporating the residual values at the end of this period.

Fernandez proposes the answers for the case of a perpetuity (we will briefly recall his argument in Section 1). In particular, he obtains a formula linking together the costs of capital of the ingredients. Then he proposes to use this formula for the general case. This particular move raises some doubts as to its validity.

Here we propose a method based on the same idea as for the perpetuity but applied to a general single step. For the multi-step general case the valuation is recursive, going backwards in time. This method is different from all 23 theories presented in Fernandez [F2]. We illustrate the formulas by examples comparing the values with those obtained by the method of Fernandez.

1. Perpetuities

We will give a brief account of the argument presented in [F1].

We assume that the depreciation each year is at the same level as the capital investment. The crucial relation is concerned with two ways of decomposing the total value of the firm (which is the present value of the total generated cash) as seen by debt holders, shareholders and the government and on the other hand shareholders and government in a hypothetical situation of an unlevered firm with the same operations:

$$V_{\text{total}} = D + E + G = V_u + G_u .$$

Since the amount of tax is proportional to the cash flow to equity, these two flows bear the same risk and the same discount rate should be applied. The relation $\text{Tax} = \frac{T \times CF}{1-T}$ carries over to the present values hence $G = \frac{T \times E}{1-T}$ and so $G + E = \frac{E}{1-T}$. The same is true, as far the free cash flow is concerned, with the tax paid by an unlevered firm being $\text{Tax}_u = \frac{T \times FCF}{1-T}$ hence $G_u = \frac{V_u}{1-T}$ and $V_{\text{total}} = \frac{V_u}{1-T}$. These relations yield

$$D(1-T) + E = V_u$$

This implies $PVTS = DT$ and also the relationship between the costs of capital

$$k_u = \frac{D(1-T)}{V_u} k_D + \frac{E}{V_u} k_E .$$

The cost of capital k_u does not change when the financial structure is altered and it is related to the operational risk of the firm only. We assume for simplicity that neither does the cost of debt, so the risk concerned with the presence of debt is reflected in the cost of equity by

$$k_E = k_u + \frac{D(1-T)}{E} (k_u - k_D) .$$

This formula, valid for perpetuities is then applied by Fernandez to a general multi-step case, where he allows general cash flows.

Example 1.

We assume that a company is operating as a perpetuity from year 2 onwards and we find its value at the end of year one. This will give us the residual value for the analysis of the single step remaining to reach the present time. Assume that the operation profit is constant for years 2,3... at the level of $OP = 400$. With annual depreciation of 60 this gives $EBIT = 340$. The debt is constant at $D = 250$ with the cost of debt $k_D = 8\%$. Subtracting 20 of interest and then 25% of taxes gives $EAT = 240$, and since we are investing 60 each year, cash flow to equity is the same $CF = 240$. For the unlevered company, the tax is 25% of $EBIT$, so the free cash flow is $FCF = 255$. Assume that the cost of capital (unlevered) is $k_u = 16\%$. We can easily compute all the ingredients in the valuation scheme.

$$V_u = \frac{255}{k_u} = 1593.75$$

$$G_u = \frac{85}{k_u} = 531.25$$

$$V_{\text{total}} = 2125$$

$$PVTS = DT = 62.50$$

$$E = V_u + PVTS - D = 1406.25$$

$$k_E = 17.0667\%$$

$$G = V_{\text{total}} - E - D = \frac{\text{tax}}{k_E} = 468.75$$

2. Single step

We compute the value of all components at time t on the basis on known expected cash flows and the values at the end of year $t+1$. For notational simplicity we set $t=0$ and the general relation is as before

$$V_t(0) = D(0) + E(0) + G(0) = V_u(0) + G_u(0).$$

We assume that the cost of debt $k_D(0)$ and cost of unlevered company $k_u(0)$ are given. The formula for the cost of equity $k_E(0)$ will be derived below. We assume, that the values $V_t(1), D(1), E(1), G(1), V_u(1), G_u(1)$ are known.

We identify some groups of cash flows at the end of year one

1. $C_t(1)$ – the total cash flow composed of the cash generated by the company together with the terminal value $V_t(1)$, discounted by $k_u(0)$ and giving $V_t(0)$,
2. C_D – the cash for debt holders composed of the interest, change of debt, and the value of debt $D(1)$, discounted at $k_D(0)$,
3. C_E – the cash for the shareholders composed of the cash flow at the end of the year and the value $E(1)$, discounted by $k_E(0)$,
4. C_G – the cash for the government (taxes) including the value $G(1)$ also discounted by $k_E(0)$ (the taxes are proportional to the cash flow to the shareholders so the returns are the same and so is the risk).

The inclusion of the terminal value in the cash available is justified since for instance the shareholders can sell the shares and debt holders can sell the bonds, except for the government that is regarded as an investor in an abstract sense.

Application of the basic idea of portfolio theory, regarding the company as a portfolio of debt, equity and government, results in the following relation

$$(*) \quad \frac{C_D(1)}{1+k_D(0)} + \frac{C_E(1)+C_G(1)}{1+k_E(0)} = \frac{C_t(1)}{1+k_u(0)}.$$

In this formula the only unknown quantity is the cost of equity, and so it can be computed.

To complete the analysis it is now sufficient to give the formulae for the numerators in (*) (which is elementary finance):

$$\begin{aligned}
C_D(1) &= k_D(0) \times D(0) + D(1) + (D(0) - D(1)) \\
C_E(1) &= E(1) + G(1) + (EBIT(1) - k_D(0) \times D(0)) \times (1 - T) \\
&\quad + (D(1) - D(0)) - \text{Investment}(1) + \text{Depreciation}(1) \\
C_G(1) &= G(1) + (EBIT(1) - k_D(0) \times D(0)) \times T \\
C_I(1) &= EBIT(1) - \text{Investment}(1)
\end{aligned}$$

Example 2.

The values found above for the end of year 1 are supplemented with the cash flow generated during year one. Assume that the operational profit is the same as for the subsequent years, i.e. $OP(1) = 400$, depreciation is 50 with slightly higher investment equal to 80. The debt at the beginning is assumed to be $D(0) = 300$ so the reduction of its level puts a burden on the cash flow to equity. We have $EBIT = 350$, unlevered tax 87.50, $FCF = 232.50$, which together with $V_u(1) = 1593.75$ and $G_u(1) = 531.25$ give the total cash on the right of equation (*) equal to 2445.

Taking account of the interest 24, taxes 81.50, change of debt level and the investment, we have $C_E(1) + C_G(1) = 164.50$. This together with the value of equity $E(1) = 1406.25$ and the value of future taxes $G(1) = 468.75$ gives the total amount, which has to be discounted by the cost of equity $k_E(0)$. With the initial debt known $D(0) = 300$ (or, equivalently, computed independently by discounting the interest 24, the sum 50 of debt reduction and $D(1) = 250$ at the rate $k_D(0) = 8\%$) we have the relation

$$300 + \frac{2121}{1 + k_E(0)} = \frac{2445}{1 + 16\%}.$$

Hence we can find

$$\begin{aligned}
k_E(0) &= 17.3276\% \\
E(0) &= \frac{164.50 + 1406.25}{1 + k_E(0)} = 1338.77 \\
V(0) &= D(0) + E(0) = 1638.77 \\
V_u(0) &= 1574.35 \\
PVTS &= 64.42
\end{aligned}$$

The cost of equity is higher than before due to the higher level of debt at time zero.

Comparison.

Using the approach of Fernandez, the values obtained are slightly different. We begin with the present value of tax shield, which according to [F2] is

$$PVST = \frac{k_u \times T \times D(0) + PVST(1)}{1 + k_u} = 64.22$$

(where of course $PVTS(1) = D \times T$, since our example assumes a perpetuity later on). The value of the equity can be found on the basis of the above present value of tax shield or, independently, by solving the following system of equations

$$\begin{cases} k_E(0) = k_u(0) + \frac{(1-T)D(0)}{E(0)}(k_u(0) - k_D(0)) \\ E(0) = \frac{C_E(1) + E(1)}{1 + k_E(0)} \end{cases}$$

which yields

$$\begin{aligned} k_E &= 17.3447\%, \\ E(0) &= 1338.58. \end{aligned}$$

Remark.

We assumed that the cost of capital for an unlevered company is given. However, one may take the initial cost of equity as known. For instance, it may result from applying CAPM and historical data. In such a case we treat the value of the cost k_u as a working assumption and after finding k_E , we adjust k_u so that k_E reaches the required level. Using a spreadsheet and goal seek tool this is straightforward, whereas an analytic formula, though possible to derive, would be very complicated.

Note: The Excel files with the details of the examples above and an example of a multi-stage complete computation are supplied in the electronic version of the paper to be found in the web edition of the journal.

Bibliography

[BS] S.Benninga, O.Sarig, *Corporate Finance: A valuation Approach*, McGraw-Hill, 1996.

[F1] P.Fernandez, *Valuation and Shareholder value Creation*, Academic Press, San Diego 2002.

[F2] P.Fernandez, *The correct value of tax shields. An analysis of 23 theories*, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=276051